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Modelling of scattering of sound from trees by the PSTD method

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Summary
In an urban environment with multiple building façade reflections, scattering from trees will increase the diffusivity of the sound field and can lead to a reduction of the noise level. Here, the possibility to incorporate scattering objects as the trunk and branches of trees in the framework of an urban propagation model, the Fourier pseudospectral time-domain (PSTD) method, has been investigated. Two different modelling approaches are studied, with either a different density of the scattering objects or with the acoustic normal velocity components at the scattering boundaries and all acoustic components inside the scattering volumes equal to zero. The first PSTD scattering approach shows to work best, although some underestimation of levels is shown. The accuracy of modelling scattering from trees by using a coarser mesh is investigated.

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1. Introduction

Despite its possible relevance with respect to combat noise from surface transport, application of vegetation in the built environment in order to reduce noise propagation is a subject which has received main attention only recently [1]. In urban environments characterized by narrow street canyons, examples of such vegetation are bushes placed at building roofs bordering a roadside courtyard to reduce the noise propagating to these areas by scattering and absorption and trees placed in streets and roadside courtyards to increase the diffusivity of the sound field. Sound propagation to roadside courtyards from nearby streets has been quantified by accurate models as BEM, ESM, FDTD and PSTD, see [2] for references, of which the latter two models have the advantage to be able to include inhomogeneous media effects as wind and temperature gradients and also model small scale scattering objects. Sound scattering from trees has not been incorporated by these prediction models so far. In this work, two modelling approaches of scattering of sound from a tree structure by the PSTD method are proposed

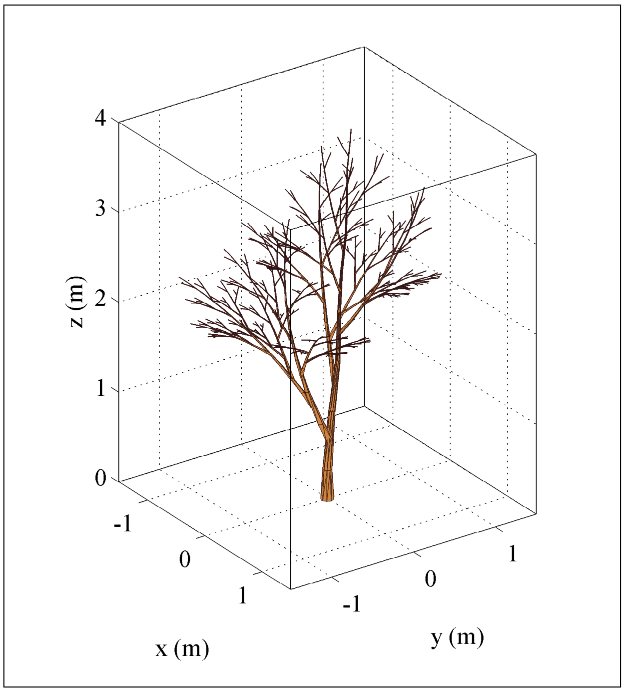

Figure 1. The studied tree structure on a rigid ground surface.

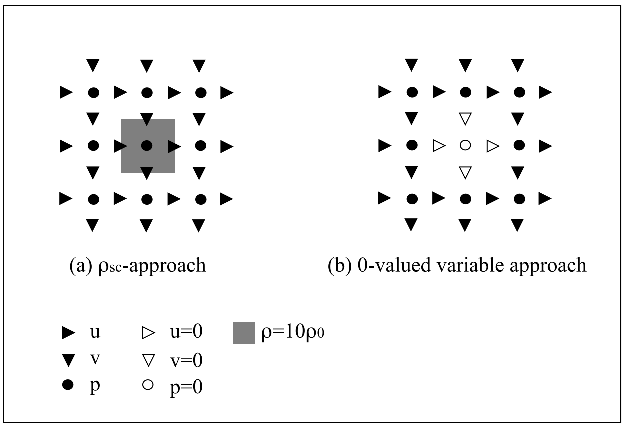

Figure 2. Illustration of the two approaches to incorporate a rigid scatterer in the PSTD grid. Samples of the spatial PSTD grid are shown with staggered positions of velocity and pressure nodes.

and validated, and ways to incorporate tree scattering in further calculations of 3D urban configurations are addressed.

2. Modelling approaches

2.1. Concepts

The Fourier pseudospectral time-domain method, here further denoted as PSTD, enables to compute sound propagation in an urban configuration by solving the following linearized Euler equations:

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u}_0 \cdot \nabla)\mathbf{u} - (\mathbf{u} \cdot \nabla)\mathbf{u}_0 - \frac{1}{\rho_0}\nabla p,$$
$$\frac{\partial p}{\partial t} = -\mathbf{u}_0 \cdot \nabla p - \rho_0 c^2 \nabla \cdot \mathbf{u}, \tag{1}$$

with $\mathbf{u}$ the velocity components, p the pressure, ρ the density and c the adiabatic sound speed. The 0-subscripted variables denote the ambient quantities. The PSTD is a domain discretization method based on an orthogonal equidistant grid. The method computes the temporal derivatives of Eqs. (1) by a Runge-Kutta method and the spatial derivatives are evaluated using a pseudospectral (PS) method, see e.g. [2]. The latter entails that only two spatial points are necessary to evaluate the smallest wavelength of interest. As a results, some 3D configurations have been studied, whereas models with a similar accuracy previously focussed on 2D configurations. The drawbacks of the PSTD method as presented in [2] are that boundaries have only been modelled either being rigid or by a second medium with a different density. For urban configurations, where horizontal zig-zag reflections prevail, this approach was found to only lead to small errors compared to modelling the boundary media as normally reacting. Also, the different density approach implies that no frequency dependent boundary reflection can be modelled by a single time domain computation. Another limitation of the PSTD method, which it shares with other orthogonal grid discretization methods, is the staircase approximation of modelling boundaries.

A simple way to include scattering objects as the tree trunk and tree branches (the boughs and twigs) in the PSTD method is to assign spatially dependent c or ρ_0 values. Such an approach has several disadvantages:

- Large spatial jumps in c or ρ_0 lead to spatial jumps in the acoustical variables and to Gibbs phenomenon corresponding to a high frequency error;
- To model acoustically rigid media, either c, ρ_0 or both should be set large. As visible from Eqs. (1), a large ρ_0 leads to a singularity. Further, a large c leads to stringent CFL condition that will likely not be met;
- Modelling a medium by a different c or ρ_0 using Eqs. (1) results in internal resonances which are radiated from the media. These effects are physically not present when the media are assumed to be rigid.

With these limitations in mind, a first scattering approach is here studied by assigning a different density to the grid positions corresponding to the scattering objects. The density is chosen such that a high reflection factor is obtained but not too high to prevent large Gibbs phenomena effects and instability. This first modelling approach is depicted in Fig. 2(a) and is further denoted as the ρ_{sc}-approach.

A second way to include scattering objects in PSTD is to strongly impose the normal acoustic velocity components at the boundaries of the scattering objects to be zero, as well as all acoustic variables inside the objects. This approach will likely give rise to errors as the values of the acoustical field variables will experience jumps across object boundaries. However, in contrast to changes in the medium properties c or ρ_0, this approach will not be sensitive to singularity issues and internal resonances can be avoided. This second PSTD scattering approach is illustrated in Fig. 2(b) and called the 0-valued variable approach.

2.2. Validation

When applying the PSTD method to model scattering from small rigid bodies in the two ways as described above, two important sources of error can be distinguished as compared to previous applications of the PSTD method in [2, 3], where errors from the spatial derivative operator, the time integration methodology, source function and implemented PML were discussed:

- The approximation of either spatially changing the density or of forcing acoustical variables to zero to model a rigid scatterer;
- The staircase approximation of the scattering object. As scatterers of small dimensions are modelled, only few grid cells (with a minimum of 1)

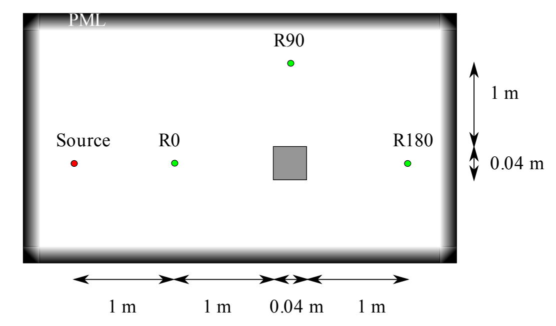

Figure 3. 2D configuration for computations with PSTD and FDTD method of scattering from a square rigid object in free field. Reflection free termination introduced by a PML.

will be used to represent the scatterer. Corners of objects are not explicitly modelled in equidistant-domain discretization methods and this error is assumed to decrease with the number of grid cells per scatterer. This source of error is expected to be similar to other equidistant domain discretization methods as the FDTD.

Two-dimensional calculations will give insight in these errors. Figure 3 shows the simulation domain, with the location of source, receiver positions and a square scatterer. The scatterer has dimensions of 0.04 m × 0.04 m. Calculations are made with both the PSTD and FDTD method and with discretizations $\Delta x = 0.04/a$ m, with $a = 1, 2, 4$. The two different ways to include a scattering object in PSTD are investigated. In the ρ_{sc}-approach, the discrete scattering boundaries are located at velocity nodes and the density at and inside the scatterer is set to $\rho_{sc} = 10\rho_0$. The implemented FDTD method also solves the linearized Euler equations (1) and has a second order accuracy both in time and space. In the FDTD method, the usual approach of setting normal velocity components at rigid object boundaries to zero has been taken. At the outer domain boundaries, a reflection free termination is obtained by a PML layer. The scattered level is computed as:

$$
\begin{aligned}
L_{sc} &= 20\log_{10}\left|\frac{|P_{tot}(f) - P_{free}(f)|}{|P_{free}(f)|}\right|, \quad (2)\\
P_{tot}(f) &= \mathcal{F}\left(p_{tot}(t)\right),\\
P_{free}(f) &= \mathcal{F}\left(p_{free}(t)\right),
\end{aligned}
$$

with $\mathcal{F}$ the forward Fourier transform, p_{tot} is the computed pressure at a receiver position in the presence of the scatterer and p_{free} the computed pressure at the same receiver position in the absence of the scatterer. Figure 4 shows L_{sc} as a function

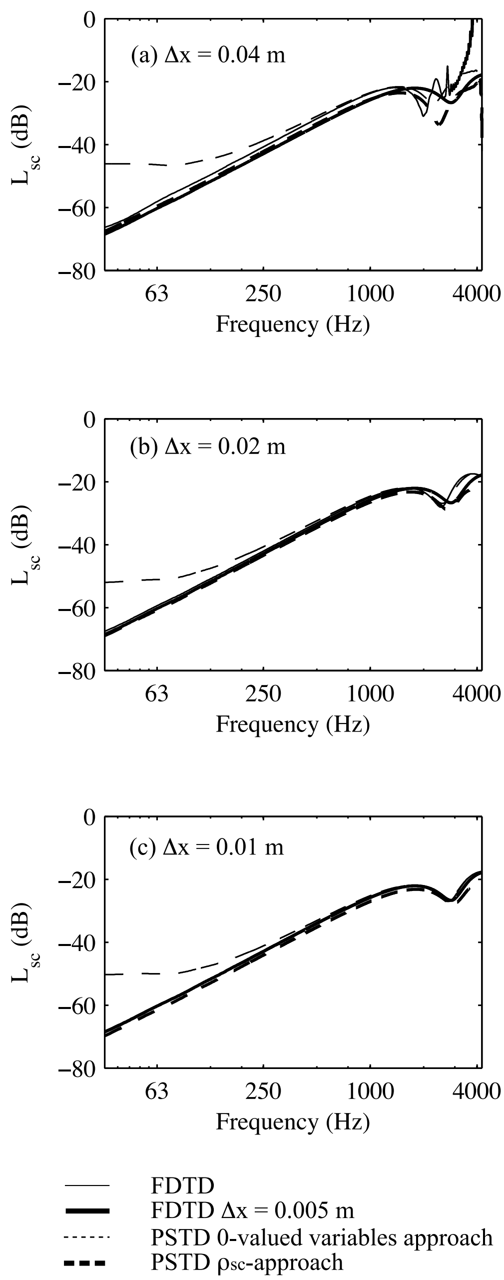

Figure 4. Scattered levels computed with FDTD and the two proposed PSTD approaches for the configuration of Fig. 3 and for receiver R0.

of the frequency up to 4 kHz for the R0 receiver position of Fig. 3 and for the 3 discretization levels. Conclusions from these results were found to also hold for the other receiver positions from Fig. 3.

Reference results from a FDTD calculation with $\Delta x = 0.005$ m are included in each plot. The results show that the level scattered from the small object logarithmically increases with frequency up to the frequency where interference effects commence (here

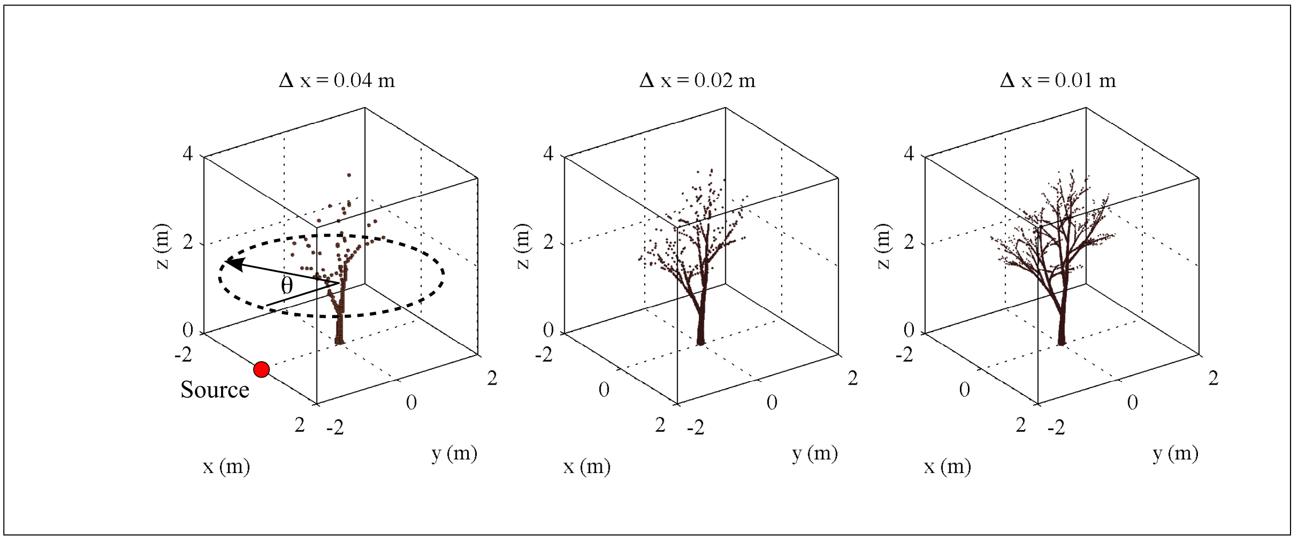

Figure 5. Discretized tree structure of Fig. 1 for three discretization levels in the PSTD method. Source position at (0,-2,0) and receiver positions at $(-2\sin\theta, -2\cos\theta, 1.5)$.

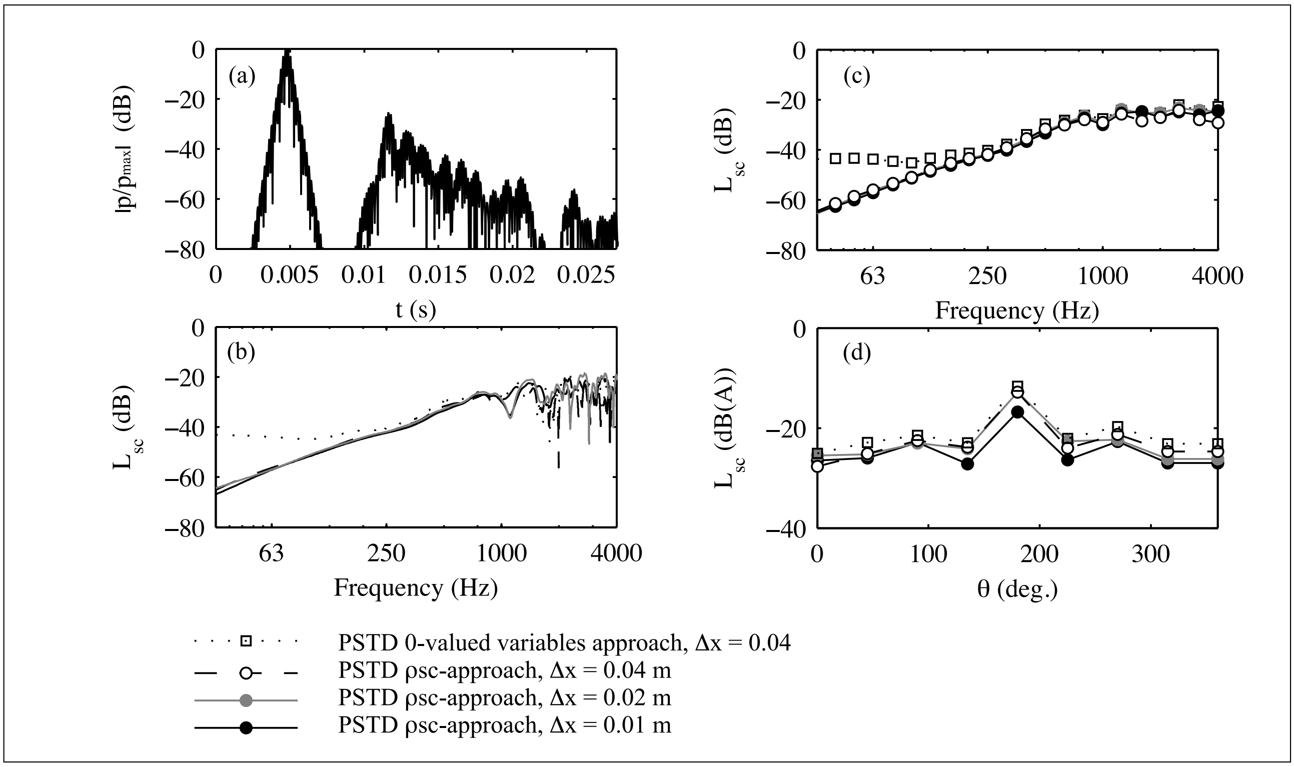

Figure 6. Scattered levels L_{sc} computed with PSTD scattering method and ρ_{sc}-approach with three levels of discretization, $\Delta x = 0.01$, $\Delta x = 0.02$ and $\Delta x = 0.04$, and PSTD method and 0-valued variable approach with $\Delta x = 0.04$. (a) Energy-time curve for receiver position $(0, -2, 1.5)$ and 2000 Hz octave band; (b) Small band levels for receiver position $(0, -2, 1.5)$; (c) 1/3-octave band levels for receiver position $(0, -2, 1.5)$; (d) Broadband levels as a function of θ and a pink noise source spectrum.

from around 3 kHz corresponding to the wavelength with twice the dimension of the diagonal length of the square). For $\Delta x = 0.04$ m, both FDTD and PSTD results clearly deviate from the reference results. The FDTD and PSTD ρ_{sc}-approach results are similar up to 1500 Hz, above which they deviate. This can be attributed to the fact that FDTD needs more points to resolve the smallest wavelength of interest compared to the PSTD method. The FDTD and PSTD ρ_{sc}-approach results for $\Delta x = 0.02$ m and $\Delta x = 0.01$ m converge to the reference solution, i.e. the staircase approximation as present in both methods reduces. The PSTD ρ_{sc}-approach slightly underpredicts the levels for all frequencies, which is caused by the non-rigid assumption of the object, i.e. $\rho_{sc} = 10\rho_0$. The ρ_{sc}-approach shows to be a rather accurate way to model scattering for the frequency range envisaged in the current geometry. An inspection of the ρ_{sc}-approach results for frequencies higher than 4 kHz indeed showed expected interference effects from resonances in the scatterer. The PSTD 0-valued variable approach converges to the reference results for the higher frequency region with a finer discretization, i.e. above 500 Hz, but deviates heavily for the lower frequencies.

3. Modelling sound scattering from trees

To efficiently model trees using the PSTD method in a setting of street canyons, it would be favourable to not have to use a finer mesh than necessary for resolving the highest frequency of interest. It is therefore relevant to investigate the accuracy of modelling scattering from trees by using various PSTD discretization levels. The two PSTD scattering approaches from the former section are therefore applied to compute scattering from a discretized tree structure on a rigid ground surface for different grid discretization levels. A tree structure is obtained from a digital library [4] and is visualized in Fig. 1. The tree is rather small compared to possible tree dimensions in an urban street and has a trunk diameter of about 0.12 m. From the vertices and faces of the tree structure, a computational algorithm has been used to determine the nodes of the orthogonal mesh as used in PSTD computations which are inside the volume of the tree. For three different mesh spacings, Fig. 5 displays the tree meshes. Due to the algorithm used, the three meshes do not necessarily lead to equal volumes of the tree structure. A point source is located at $(-2, 0, 0)$ and receivers at $(-2\sin\theta, -2\cos\theta, 1.5)$. Figure 6(a) shows the 2000 Hz octave band energy-time curve for receiver position (0,-2,1.5) computed with $\Delta x = 0.01$ m and the ρ_{sc}-approach, displaying the direct arrival and the reverberant response due to scattering from the tree structure. Results for receiver position (0,-2,1.5) are shown in small bands in Fig. 6(b). The scattered levels logarithmically increase with frequency up to the frequency where interference effects appear, i.e. around 1 kHz. For the PSTD 0-valued variable approach, calculations are only carried out with $\Delta x = 0.04$ m. The results from the PSTD ρ_{sc}-approach and the 0-valued variable approach deviate for the low frequency region which is in line with results from the previous section. The deviations among the three PSTD ρ_{sc}-approach results are small below 1 kHz but interference effects do not collapse as expected from Section 2.1 and the difference in the meshes. The 1/3-octave band results as presented in Fig. 6(c) also show largest deviations for the higher frequency region among the ρ_{sc}-approach results. Figure 6(d) finally shows the broadband L_{sc} levels in dB(A) for the various receiver positions. A pink noise source spectrum is taken. The larger L_{sc} value for the 180° position is caused by the shielding of the receiver from the direct pressure pulse. The results exhibit similar trends for all calculations but also clear level differences.

4. Conclusions and further work

For studying the effect of sound scattering from trees in an urban configuration, the possibility to model scattering objects as the trunk and branches of trees in the framework of a numerical method, the PSTD method, has been investigated. Two different modelling approaches are proposed, in which the scattering volume is discretized with 1) a different density, 2) the acoustic velocity components at the boundaries and all acoustic components inside the volume set to zero. The second approach works well for higher frequencies, yet fails the low frequency region. The first PSTD scattering approach shows to work quite well, although some lower levels are obtained by the choice of the object density. The proposed methods are used to model sound scattering from a discretized tree structure, and various discretization levels have been tested. Scattering levels for the investigated single tree geometry show a similar receiver position dependency but clear level differences.

As the chosen approach treats trees as a scatterer, it should well be possible to apply the methodology to model other vegetation as bushes and shrubs. Also, it can be noticed that the FDTD method will be suitable for accurately modelling scattering from a the fine tree structure. In the most promising PSTD scattering approach, i.e. the different density approach, a choice for the different density was made. For further work, it is recommended to find an optimal value for this density. A too high density value leads to better results for most frequencies yet to aliasing effects and instability. These artifacts could possibly be suppressed by spatially filtering the density. Current work only concerns numerical results. Comparison with measurements should be made to further assess the proposed methodology. In the current paper, a 3D tree is modelled. The necessity of modelling sound scattering from trees in 3D over a 2D approach needs further study. In further work, more tree structures will be studied to compare the amount of sound scattering among various tree species.

The modelled tree shows a low scattered level, but it might however be significant in an urban environment with multiple façade reflections, where the scattering could increase the diffusiveness of the sound field. To model tree scattering in an urban environment, the recent development of a multi-domain PSTD method is of high interest [5]. With this method, the volume containing the scattering object can be modelled by a fine grid surrounded by a mesh with a discretization as coarse as necessary to resolve all wavelength of interest. As such, local refinement is obtained and computational efficiency improved.

Acknowledgement

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